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Linear perturbations in brane gas cosmology

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Abstract: We consider the effect of string inhomogeneities on the time dependent background of Brane Gas Cosmology. We derive the equations governing the linear perturbations of the dilaton-gravity background in the presence of string matter sources. We focus on long wavelength fluctuations and find that there are no instabilities. Thus, the predictions of Brane Gas Cosmology are robust against the introduction of linear perturbations. In particular, we find that the stabilization of the extra dimensions (moduli) remains valid in the presence of dilaton and string perturbations.

Keywords: Bosonic Strings, Physics of the Early Universe.

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1. Introduction

Understanding the behavior of strings in a time dependent background has been a subject of much interest and has been pursued in a number of differing ways. One scenario, known as Brane Gas Cosmology (BGC) is devoted to understanding the effect that string and brane gases could have on a dilaton-gravity background in the early Universe [1]–[6]. In [1], it was suggested that the energy associated with the winding of strings around the compact dimensions would produce a confining potential for the scale factor and halt the cosmological expansion.[1] The analysis of BGC was initially performed at the level of the equations for homogeneous and isotropic cosmology. The results were recently extended to the case of anisotropic cosmology in [4]. There, it was shown that string gases can result in three dimensions growing large and isotropic due to string annihilation while the other six dimensions remain confined. In [5] it was shown that by considering both momentum and winding modes of strings, the six confined dimensions can be stabilized at the self-dual radius, where the energy of the string gas is minimal. This result demonstrated that, in BGC, the volume moduli of the extra dimensions can be stabilized in a natural and intuitive way.

To date, however, all analyses have been performed at the level of the equations for homogeneous cosmology. The string sources are usually represented by a perfect fluid with homogeneous energy and pressure densities given by the mass spectrum of the strings (see e.g. [3, 5]). One may worry that inhomogeneities of string sources (in particular strings winding around the confined dimensions) as a function of the unconfined spatial directions could lead to serious instabilities which would ruin one of the main successes of BGC, namely the prediction that three directions become large, leaving the other six confined uniformly as a function of the coordinates of the large spatial sections.

[1] This was later shown quantitatively in [2].

As a first step towards addressing this concern, we here study the evolution of linear fluctuations about a BGC background. We introduce linear matter and metric fluctuations, derive the linear perturbation equations, and study their stability on long wavelength (wavelengths larger than the Hubble radius).[2] Note that since the dilaton plays a vital role in the stabilization process, we must also consider the effect of inhomogeneities in the dilaton field. Our main result is that the long wavelength fluctuations do not grow in time — there are no instabilities at the level of a linearized analysis.

In section 2, we present the equations of BGC and review the background solution that leads to $3+1$ dimensions growing large and 6 dimensions being stabilized at the self-dual radius. In section 3, we introduce the fluctuations and derive the equations for linearized perturbations about the time dependent homogeneous background of BGC. The long wavelength solutions are given in section 4, leaving a detailed description for the appendix. We conclude with a discussion of the results in section 5.

2. Background solution

Our starting point is the low energy effective action for the bulk space-time with string matter sources [2],

$$S = \frac{1}{4\pi\alpha'} \int d^D x \sqrt{-g} e^{-2\varphi} \left(R + 4(\nabla\varphi)^2 - \frac{1}{12} H^2 \right) + S_{\rm m} \,, \tag{2.1}$$

where R denotes the Ricci scalar, g is the determinant of the background metric, φ is the dilaton field, and H is the field strength of an antisymmetric tensor field. The action of the matter sources is denoted by $S_{\rm m}$. For example, with $D = 10$ this is the low energy effective action of type II-A superstring theory. For the purposes of this paper we will ignore the effects of branes, since it will be the winding and momentum modes of the string that ultimately determine the dimensionality and stability of space-time [3]. Here, we will ignore the effects of fluxes,[3] i.e. we set $H = 0$.

This action yields the following equations of motion,

$$\begin{aligned} R_\mu{}^\nu + 2\nabla_\mu \nabla^\nu \varphi &= 8\pi M_p^{-2}\, e^{2\varphi} T_\mu{}^\nu \,, \\ R + 4\nabla_\kappa \nabla^\kappa \varphi - 4\nabla_\kappa \varphi \nabla^\kappa \varphi &= 0 \,, \end{aligned} \tag{2.2}$$

where ∇ is the covariant derivative.

We will work in the conformal frame with a homogeneous metric of the form

$$ds^2 = e^{2\lambda(\eta)} \left(d\eta^2 - \delta_{ij} dx^i dx^j \right) - e^{2\nu(\eta)} \delta_{mn} dx^m dx^n \,, \tag{2.3}$$

where (η, x^i) are the coordinates of $3+1$ space-time and x^m are the coordinates of the other six dimensions, both of which can be taken to be isotropic [4]. The scale factors $a(\eta)$ and $b(\eta)$ are given by $\lambda \equiv \ln(a)$ and $\nu \equiv \ln(b)$.

[2] On shorter wavelengths, we expect that the motion of the strings will smear out potential instabilities in a way analogous to how the motion of light particles ("free-streaming") leads to a decay of short wavelength fluctuations in standard cosmology (see e.g. [7] for a review).

[3] See [8] for inclusion of fluxes in the scenario.

We consider the effect of the strings on the background through their stress energy tensor

$$T_{\mu}{}^{\nu} \equiv \mathrm{diag}(\rho, -p_i, -p_m)\,, \tag{2.4}$$

where ρ is the energy density of the strings, p_i ($i = 1, \ldots, 3$) is the pressure in the expanding dimensions and p_m ($m = 4, \ldots, 9$) is the pressure in the small dimensions (because of our assuption of isotropy of each subspace, there is only one independent p_i and one independent p_m).

Strings contain winding modes, momentum modes and oscillatory modes. However, since the energies of the oscillatory modes are independent of the size of the dimensions, and since the winding modes and momentum modes dominate the thermodynamic partition function at very small and very large radii of the spatial dimensions, we shall here neglect the oscillatory modes. In the absence of string interactions, the contributions to the stress tensor coming from the string winding modes and momentum modes ($T^{\mathrm{w}}_{\mu\nu}$ and $T^{\mathrm{m}}_{\mu\nu}$ respectively) are separately conserved,

$$\begin{aligned} T_{\mu\nu} &= T^{\mathrm{w}}_{\mu\nu} + T^{\mathrm{m}}_{\mu\nu} \\ \nabla^{\mu} T^{\mathrm{w}}_{\mu\nu} &= 0 \qquad \nabla^{\mu} T^{\mathrm{m}}_{\mu\nu} = 0\,. \end{aligned} \tag{2.5}$$

The conservation equations take the form

$$\rho'^{\,\mathrm{w,m}} + \sum_{i=1}^{9} \lambda_i' (\rho^{\mathrm{w,m}} - p_i^{\mathrm{w,m}}) = 0\,, \tag{2.6}$$

where the derivatives are with respect to the conformal time η, and where for the moment we consider 9 independent scale factors.

Expressing (2.2) in terms of the metric (2.3) and the stress tensor (2.4), we find the following system of equations,

$$-3\lambda'' - 6\nu'' + 6\lambda'\nu' - 6\nu'^2 + \varphi'' - \lambda'\varphi' = 8\pi M_p^{-2} e^{2\varphi+2\lambda}\rho \tag{2.7}$$
$$-\lambda'' + 2\lambda'^2 + 6\lambda'\nu' + \lambda'\varphi' = -8\pi M_p^{-2} e^{2\varphi+2\lambda} p_i \tag{2.8}$$
$$-\nu'' + 6\nu'^2 + 2\lambda'\nu' + \varphi'\nu' = -8\pi M_p^{-2} e^{2\varphi+2\lambda} p_m \tag{2.9}$$
$$\begin{aligned} -6\lambda'' - 12\nu'' - 24\lambda'\nu' - 42\nu'^2 - 6\lambda'^2 - \varphi'^2 + \\ +2\varphi'' + 8\lambda'\varphi' + 12\varphi'\nu' = 0\,. \end{aligned} \tag{2.10}$$

The explicit forms of the energy density and pressure were given in [5],[4]

$$\rho = 3\mu N^{(3)} e^{-2\lambda-6\nu} + 3\mu M^{(3)} e^{-4\lambda-6\nu} + 6\mu N^{(6)} e^{-3\lambda-5\nu} + 6\mu M^{(6)} e^{-3\lambda-7\nu}\,, \tag{2.11}$$
$$p_i = -\mu N^{(3)} e^{-2\lambda-6\nu} + \mu M^{(3)} e^{-4\lambda-6\nu}\,, \tag{2.12}$$
$$p_m = -\mu N^{(6)} e^{-3\lambda-5\nu} + \mu M^{(6)} e^{-3\lambda-7\nu}, \tag{2.13}$$

where μ is a constant, $N^{(3)}$ and $M^{(3)}$ are the numbers of winding and momentum modes in the large directions, and $N^{(6)}$ and $M^{(6)}$ in the six small directions.

[4]The equations here are related to eq. (18) in [5] by the volume factor $V = e^{3\lambda+6\nu}$, e.g. $\rho = E/V$.

We are interested in solutions that stabilize the internal dimensions, while allowing the three large dimensions to expand. Such solutions were discussed in [5], where it was shown that the winding and momentum modes of the strings lead naturally to stable compactifications of the internal dimensions at the self dual radius, while the other three dimensions grow large since the string modes winding them have annihilated, allowing for expansion. Thus, we will set $N^{(3)} = 0$. At the self dual radius, the number of winding modes is equal to the number of momentum modes (i.e. $N^{(6)} = M^{(6)}$) and the pressure vanishes ($p_m = 0$).

In ref. [5], the solutions subject to the above conditions on the winding and momentum numbers were found numerically. In this paper, we wish to study the stability of these solutions towards linear perturbations in the time interval when the internal dimensions have stabilized and the large dimensions give power law expansion. In the following section, we will derive the equations for the linear fluctuations. The coefficients in these equations depend on the background solution. We will use analytical expressions which approximate the numerically obtained solutions of [5]. We restrict our initial conditions so that the evolution preserves the low energy and small string coupling assumptions ($g_s \sim e^{2\varphi} \ll 1$).

A approximation to a typical solution of the equations (2.7)–(2.10) is found to be of the form

$$\begin{aligned} \lambda(\eta) &= k_1 \ln(\eta) + \lambda_0 \qquad \text{or} \qquad a(\eta) = a_0 \eta^{k_1}\,, \\ \varphi(\eta) &= -k_2 \ln(\eta) + \varphi_0\,, \end{aligned} \tag{2.14}$$

where the constants k_1, k_2, λ_0 and φ_0 depend on the choice of initial conditions. We have made use of $\nu = \nu' = \nu'' = 0$, $N^{(3)} = 0$, $N^{(6)} = M^{(6)}$, $p_m = 0$. Note that in this limit (2.9) is trivially satisfied. An example of a solution yielding stabilized dimensions and three dimensions growing large corresponds to $k_1 = 1/9$ and $k_2 = 9/7$. The numerical solution of [5] and the analytical approximation used in this paper are compared in figure 1, for the above values of the constants k_1 and k_2.

3. Scalar metric perturbations

In this section we consider the growth of scalar metric perturbations (see e.g. [9] for a comprehensive review of the theory of cosmological perturbations) due to the presence of string inhomogeneities. We are interested in the case where the fluctuations depend only on the external coordinates and conformal time, not on the coordinates of the internal dimensions. For simplicity we work in the generalized longitudinal gauge in which the metric perturbations are only in the diagonal metric elements.[5] Thus, the metric including linear fluctuations is given by

$$ds^2 = e^{2\lambda(\eta)}((1+2\phi)d\eta^2 - (1-2\psi)\delta_{ij}dx^i dx^j) - e^{2\nu(\eta)}(1-2\xi)\delta_{mn}dx^m dx^n\,. \tag{3.1}$$

[5]As discussed e.g. in [10], for scalar perturbations depending on all spatial coordinates it would be inconsistent to choose the perturbed metric completely diagonal, and one would have to add a metric coefficient to the $dtdx^m$ terms, where x^m are the coordinates of the internal dimensions. However, as discussed in [11], if the fluctuations are independent of the coordinates x^m, as in our case, the coefficient can be chosen to vanish, and thus the perturbed metric is completely diagonal.

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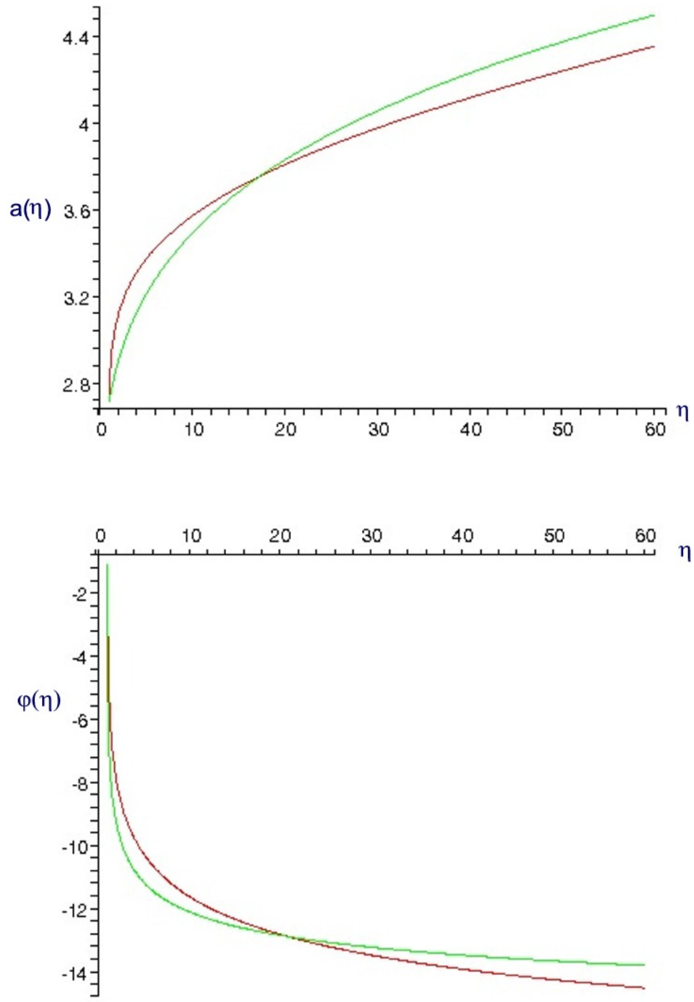

Figure 1: A comparison between the numerical background solutions obtained in [5] (red or light line) and the analytical approximation used in this paper (green or dark line).

The dilaton φ also fluctuates about its background value φ_0. The dilaton fluctuation χ is determined by

$$\varphi = \varphi_0 + \delta\varphi \qquad \chi \equiv \delta\varphi\,. \tag{3.2}$$

In the above, the fluctuating fields χ, ϕ, ψ and ξ are functions of the external coordinates x^i and time, i.e.

$$\chi = \chi(\eta, x^i)\,, \qquad \phi = \phi(\eta, x^i)\,, \qquad \psi = \psi(\eta, x^i)\,, \qquad \xi = \xi(\eta, x^i)\,. \tag{3.3}$$

The perturbations of the matter energy momentum tensor result from over-densities and under-densities in the number of strings. From (2.11)–(2.13) and noting that we are interested in the case when $N^{(3)} = 0$ and $M^{(6)} = N^{(6)}$ we find,

$$\delta\rho = \delta\rho_{\mathrm{w}} + \delta\rho_{\mathrm{m}}\,, \tag{3.4}$$

$$\delta\rho_{\mathrm{w}} = 30\mu N\xi e^{-3\lambda} + 18\mu N\psi e^{-3\lambda} + 6\delta N^{(6)} e^{-3\lambda}\,, \tag{3.5}$$

$$
\begin{aligned}
\delta\rho_{\rm m} &= 42\mu N\xi e^{-3\lambda} + 18\mu M\xi e^{-4\lambda} + 18\mu N\psi e^{-3\lambda} + 12\mu M\psi e^{-4\lambda} + \\
&\quad + 6\mu\delta M^{(6)}e^{-3\lambda} + 3\mu\delta M e^{-4\lambda}\,, && (3.6)\\
\delta p_\lambda &= 6\mu M\xi e^{-4\lambda} + 4\mu M\psi e^{-4\lambda} + \mu\delta M e^{-4\lambda}\,, && (3.7)\\
\delta p_\nu &= 2\mu N\xi e^{-3\lambda} - \mu\delta N^{(6)}e^{-3\lambda} + \mu\delta M^{(6)}e^{-3\lambda}\,, && (3.8)
\end{aligned}
$$

where we define[6] $N \equiv N^{(6)} = M^{(6)}$ and $M \equiv M^{(3)}$. The fluctuations $\delta N^{(6)}$, $\delta M^{(6)}$, and δM are taken as functions of both conformal time and the external space, e.g. $\delta N^{(6)} = \delta N(\eta, x^i)$.

It follows from (2.6) that the perturbed sources obey modified conservation equations for both the winding and momentum modes,

$$
\delta\rho'^{\,\rm w,m} + \sum_{i=1}^{9}\lambda_i'(\delta\rho^{\rm w,m} - \delta p_i^{\rm w,m}) + \sum_{i=1}^{9}\delta\lambda_i'(\rho^{\rm w,m} - p_i^{\rm w,m}) = 0\,, \qquad (3.9)
$$

where $\delta\lambda = a^{-1}\delta a = -\psi$ and $\delta\nu = b^{-1}\delta b = -\xi$ are spatial variations.

We rewrite (2.2) to take the more familiar form of the Einstein and dilaton equations, namely

$$
\begin{aligned}
R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R &= e^{2\varphi}T_\mu^\nu - 2g^{\alpha\nu}\nabla_\mu\nabla_\alpha\varphi + 2\delta_\mu^\nu(g^{\mu\nu}\nabla_\mu\nabla_\nu\varphi - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi)\,,\\
g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\nabla_\mu\nabla_\nu\varphi &= \frac{1}{4}e^{2\varphi}T_\mu^\mu\,, && (3.10)
\end{aligned}
$$

where we invoke Planckian units (i.e. $8\pi M_p^{-2} = 1$). Plugging the perturbed metric (3.1) and dilaton into these equations, making use of the background equations of motion, and linearizing the equations about the background (i.e. keeping only terms linear in the fluctuations) yields the following set of equations:

$$
\begin{aligned}
\vec\nabla^2\psi + 3\vec\nabla^2\xi - 9\mathcal{H}\xi' - 3\mathcal{H}\psi' - 3\mathcal{H}^2\phi &= \frac{1}{2}e^{2\varphi+2\lambda}(2\chi T_0^0 + \delta T_0^0) - 6\mathcal{H}\phi\varphi' - 3\psi'\varphi' - \\
&\quad - 6\xi'\varphi' - \vec\nabla^2\chi + 3\mathcal{H}\chi' + 2\phi\varphi'^2 - 2\chi'\varphi'\,, && (3.11)\\
\partial_i\psi' + 3\partial_i\xi' + \mathcal{H}\partial_i\phi - 3\mathcal{H}\partial_i\xi &= \frac{1}{2}e^{2\varphi+2\lambda}\delta T_0^i + \partial_i\phi\varphi' - \partial_i\chi' + \mathcal{H}\partial_i\chi\,, && (3.12)\\
\partial_i\partial_j(\phi - \psi - 6\xi - 2\chi) &= 0 \qquad i \neq j\,, && (3.13)\\
(\partial_i^2 - \vec\nabla^2)(\phi - \psi - 6\xi) - 2\psi'' - 6\xi'' - \\
-4\mathcal{H}\psi' - 6\mathcal{H}\xi' - 2\mathcal{H}^2\phi - 4\mathcal{H}'\phi - 2\phi'\mathcal{H} &= e^{2\varphi+2\lambda}(2\chi T_i^i + \delta T_i^i) + 2\partial_i^2\chi - 4\phi\varphi'' - \\
&\quad - 2\phi'\varphi' - 4\mathcal{H}\phi\varphi' - 4\psi'\phi' - 12\xi'\varphi' + 2\chi'' - \\
&\quad - 2\vec\nabla^2\chi + 2\mathcal{H}\chi' + 4\phi\varphi'^2 - 4\chi'\varphi'\,, && (3.14)\\
-\vec\nabla^2\phi + 5\vec\nabla^2\xi - 5\xi'' + 2\vec\nabla^2\psi - 3\psi'' - \\
-10\mathcal{H}\xi' - 3\phi'\mathcal{H} - 9\mathcal{H}\psi' - 6\mathcal{H}^2\phi - 6\mathcal{H}'\phi &= e^{2\varphi+2\lambda}(2\chi T_m^m + \delta T_m^m) - 4\phi\varphi'' - 2\phi'\varphi' - \\
&\quad - 8\mathcal{H}\phi\varphi' - 6\psi'\varphi' - 10\xi'\varphi' + 2\chi'' - \\
&\quad - 2\vec\nabla^2\chi + 4\mathcal{H}\chi' + 4\phi\varphi'^2 - 4\chi'\phi'\,, && (3.15)
\end{aligned}
$$

[6]Notice that we must be careful to distinguish between the perturbed quantities $\delta N^{(6)}$ and $\delta M^{(6)}$.

$$
\begin{aligned}
&-2\phi\varphi'^2 + 2\varphi'\chi' + \phi\varphi'' - 6\psi\mathcal{H}\varphi' + \\
&\quad +\frac{1}{2}\phi'\varphi' - 2\mathcal{H}\phi\varphi' - \frac{3}{2}\psi'\varphi' - \\
&\quad -3\xi'\varphi' - \frac{1}{2}\chi'' + \frac{1}{2}\vec{\nabla}^2\chi - \mathcal{H}\chi' = \frac{1}{4}e^{2\varphi+2\lambda}(2\chi T + \delta T)\,, \qquad (3.16)
\end{aligned}
$$

where $T \equiv T^{\mu}_{\mu}$ is the trace. The modified conservation equations (3.9) take the form

$$
\frac{d}{d\eta}(\delta N^{(6)}) = 7N\xi'\,, \qquad (3.17)
$$

$$
\begin{aligned}
&42\mu N\xi' - 72\mu M\xi\lambda' e^{-\lambda} - 48\mu M\psi\lambda' e^{-\lambda} + 12\mu M\psi' e^{-\lambda} + \\
&\quad +6\mu\frac{d}{d\eta}(\delta M^{(6)}) - 12\mu\delta M\lambda' e^{-\lambda} + 3\mu\frac{d}{d\eta}(\delta M)e^{-\lambda} = 0\,. \qquad (3.18)
\end{aligned}
$$

These equations give us the evolution of the metric perturbations ϕ, ψ, and ξ in terms of the matter perturbations χ, $\delta\rho$, and δp_i. At first glance, it may appear that the above system is over-determined since we have eight equations for seven unknowns. However, as is the case in standard cosmology, the conservation equations are not independent of the Einstein equations. Thus, we can choose to keep only one of the modified conservation equations and our system will be consistent.

4. Evolution of fluctuations

In this section we focus on the evolution of long wavelength perturbations, since instabilities on these scales would be the most dangerous for the success of BGC at providing a mechanism which allows exactly three spatial dimensions to become large and stabilizes the radius of the other dimensions at a microscopic value. We focus on the time interval $\eta \gg 1$ when the hierarchy in scales between the large and the small spatial dimensions has already developed, and where we can use the analytical approximations to the background dynamics given in section 2.

Since we are focusing on long wavelength fluctuations, we can neglect all terms with spatial gradients in Equations (3.11)–(3.16). Making use of (3.13) we can eliminate one of the scalar metric perturbations, ϕ, from the rest of the Equations (3.11)–(3.16). We take Equation (3.11) as a constraint on the initial data, noticing that it only contains first derivatives of the perturbation variables. By using the background solution (2.14) we find that the remaining equations represent a coupled system of harmonic oscillators with time dependent coefficients. With these approximations, the equations become[7]

$$
-2\chi''-2\psi''-6\xi''-\frac{230}{7\eta}\xi'-\frac{176}{21\eta}\psi'-\frac{230}{21\eta}\chi'-\frac{12868}{3969\eta^2}\chi-\frac{6434}{3969\eta^2}\psi-\frac{12868}{1323\eta^2}\xi = 0\,, \qquad (4.1)
$$

$$
-3\psi''-5\xi''-2\chi''-\frac{244}{21\eta}\psi'-\frac{1978}{63\eta}\xi'-\frac{718}{63\eta}\chi'-\frac{2672}{1323\eta^2}\psi-\frac{5344}{441\eta^2}\xi-\frac{5344}{1323\eta^2}\chi = 0\,, \qquad (4.2)
$$

$$
-\frac{1}{2}\chi''-\frac{250}{63\eta}\chi'+\frac{9}{7\eta}\psi'-\frac{170}{49\eta^2}\chi-\frac{43}{49\eta^2}\psi-\frac{510}{49\eta^2}\xi = 0\,. \qquad (4.3)
$$

[7] The full equations can be found in the appendix.

Notice that the string source terms, δN and δM, have disappeared from the equations. This is due to the fact that, for long wavelength fluctuations, the terms due to purely gravitational dynamics (e.g. the damping terms in the evolution of the radius of the internal dimensions as a consequence of the expansion of the large dimensions) are more important than the matter sources. This is not too surprising based on the results of the theory of linear cosmological perturbations [9] which show that on super-Hubble scale matter is pulled along by gravity, but that it is the purely gravitational dynamics which determines the growth rate of the fluctuations. In analogy, we find here that the terms representing the string matter sources are sub-leading in the equations (see appendix). This realization is the key physical reason which leads to our ultimate conclusion that also in BGC there are no instabilities of long wavelength fluctuations.

The above set of coupled equations can be solved exactly, and we find the leading terms to be

$$\psi \sim \frac{c_1}{\eta^{0.06}} + \frac{c_2}{\eta^{0.36}} , \tag{4.4}$$

$$\xi \sim c_3 + \frac{c_4}{\eta^{0.06}} , \tag{4.5}$$

$$\chi \sim c_5 + \frac{c_6}{\eta^{0.06}} . \tag{4.6}$$

This is our main result. First, we have found that the perturbation of the internal geometry ξ is constant to leading order. Thus, the stabilization mechanism resulting from considering winding and momentum modes remains viable in the presence of string inhomogeneities and dilaton fluctuations. Secondly, the geometry of the $3+1$ dimensions growing large does not suffer from exponential instabilities. In fact, we have power-law decay for ψ and the dilaton perturbation χ decays to a constant.

The crucial point that led to this behavior was the fact that the string source terms were sub-leading in the equations above. Physically this means that the number of winding and momentum modes moving in or out of a region is negligible compared to the expansion rate and the evolution of the perturbations. Therefore, there is an *averaging* of the long wavelength perturbations that leads to a smoothing effect and the string inhomogeneities are rendered harmless.

5. Conclusions

We have considered the evolution of linearized string inhomogeneities and dilaton fluctuations in the background of Brane Gas Cosmology (BGC). We considered fluctuations which are independent of the spatial coordinates of the internal dimensions. We have derived the perturbation equations for BGC in the longitudinal gauge and solved them in the long wavelength approximation. We have found that for long wavelengths, the effects of the string sources is subleading compared to the purely gravitational and dilaton terms in the fluctuation equations. This generalizes the result of the conventional theory of cosmological perturbations in Einstein gravity in four space-time dimensions which states that on scales larger than the Hubble radius, self-gravitational effects dominate the dynamics, and that

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the matter sources are simply dragged along by the metric. In particular, there are no instabilities of the background solution of BGC towards such long wavelength fluctuations. Thus, we find that the predictions of BGC are robust towards the effects of matter and dilaton fluctuations. In particular, the stabilization mechanism for the extra dimensions (volume modulus) remains operative in the presence of these inhomogeneities, while the $3+1$ dimensional space-time continues to grow without instabilities.

Although this is a promising result for BGC, there are still many questions to be answered. Our analysis has been limited to the linear regime, while non-linear effects may be crucial. Our linearized analysis corresponds to taking the localized string sources and smearing them out over a length scale larger than the Hubble radius to obtain a string gas description. It would be interesting to see if these results hold when considering individual strings and their interactions and in what limits the string gas approximation holds. Also, BGC offers a dynamical way to stabilize the volume modulus, but we have not addressed the issue of stabilizing the dilaton and we have also ignored the role of fluxes. We leave these questions for further investigations.

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A. Perturbation equations

In this appendix we present a detailed derivation of the solutions presented in section 4. In the long wavelength limit we can neglect gradient terms and the equations (3.11)–(3.16) reduce to

$$\begin{aligned} -9\mathcal{H}\xi' - 3\mathcal{H}\psi' - 3\mathcal{H}^2\phi &= \frac{1}{2}e^{2\lambda+2\varphi}(2\chi T^0_0 + \delta T^0_0) - 6\mathcal{H}\phi\varphi' - 3\psi'\varphi' - \\ &\quad -6\xi'\varphi' + 3\mathcal{H}\chi' + 2\phi\varphi'^2 - 2\chi'\varphi' \,, \end{aligned} \tag{A.1}$$

$$\partial_i\partial_j(\phi - \psi - 6\xi - 2\chi) = 0 \qquad i \neq j \,, \tag{A.2}$$

$$\begin{aligned} -2\psi'' - 6\xi'' - 4\mathcal{H}\psi' - 6\mathcal{H}\xi' - \\ -2\mathcal{H}^2\phi - 4\mathcal{H}'\phi - 2\phi'\mathcal{H} &= e^{2\lambda+2\varphi}(2\chi T^i_i + \delta T^i_i) + -4\phi\varphi'' - 2\phi'\varphi' - \\ &\quad -4\mathcal{H}\phi\varphi' - 4\psi'\varphi' - 12\xi'\varphi' + 2\chi'' + \\ &\quad +2\mathcal{H}\chi' + 4\phi\varphi'^2 - 4\chi'\varphi' \,, \end{aligned} \tag{A.3}$$

$$\begin{aligned} -5\xi'' - 3\psi'' - 10\mathcal{H}\xi' - 3\phi'\mathcal{H} - \\ -9\mathcal{H}\psi' - 6\mathcal{H}^2\phi - 6H'\phi &= e^{2\lambda+2\varphi}(2\chi T^m_m + \delta T^m_m) - 4\phi\varphi'' - 2\phi'\varphi' - \\ &\quad -8\mathcal{H}\phi\varphi' - 6\psi'\varphi' - 10\xi'\varphi' + 2\chi'' + 4\mathcal{H}\chi' + \\ &\quad +4\phi\varphi'^2 - 4\chi'\varphi' \,, \end{aligned} \tag{A.4}$$

$$\begin{aligned} -2\phi\varphi'^2 + 2\varphi'\chi' + \phi\varphi'' - 6\psi\mathcal{H}\varphi' + \frac{1}{2}\varphi'\phi' - \\ -2\mathcal{H}\phi\varphi' - \frac{3}{2}\psi'\varphi' - 3\xi'\varphi' - \frac{1}{2}\chi'' - \mathcal{H}\chi' &= \frac{1}{4}e^{2\lambda+2\varphi}(2\chi\mu T^\mu_\mu + \delta T^\mu_\mu) \,. \end{aligned} \tag{A.5}$$

Making use of the background solution (2.14) and (A.2) to eliminate the scalar metric perturbation ϕ, we find the following set of equations describing the evolution;

$$\begin{aligned}
&-\frac{61}{21\eta}\chi' - \frac{61}{7\eta}\xi' - \frac{88}{21\eta}\psi' + \left(-\frac{11114}{1323\eta^2} - \frac{3000}{\eta^{\frac{176}{63}}} - \frac{12000}{\eta^{\frac{169}{63}}}\right)\chi + \\
&+\left(-\frac{36000}{\eta^{\frac{169}{63}}} - \frac{9000}{\eta^{\frac{176}{63}}} - \frac{11114}{441\eta^2}\right)\xi + \left(-\frac{6000}{\eta^{\frac{176}{63}}} - \frac{5557}{1323\eta^2} - \frac{18000}{\eta^{\frac{169}{63}}}\right)\psi - \\
&-\frac{3}{\eta^{\frac{169}{63}}}\delta M^{(6)} - \frac{3}{\eta^{\frac{169}{63}}}\delta N^{(6)} - \frac{3}{2\eta^{\frac{176}{63}}}\delta M^{(3)} = 0\,, \qquad \text{(A.6)}
\end{aligned}$$

$$\begin{aligned}
&-2\chi'' - 2\psi'' - 6\xi'' - \frac{230}{7\eta}\xi' - \frac{176}{21\eta}\psi' - \frac{230}{21\eta}\chi' + \left(-\frac{2000}{\eta^{\frac{176}{63}}} - \frac{12868}{3969\eta^2}\right)\chi + \\
&+\left(-\frac{4000}{\eta^{\frac{176}{63}}} - \frac{6434}{3969\eta^2}\right)\psi + +\left(-\frac{6000}{\eta^{\frac{176}{63}}} - \frac{12868}{1323\eta^2}\right)\xi - \frac{1}{\eta^{\frac{176}{63}}}\delta M^{(3)} = 0\,, \qquad \text{(A.7)}
\end{aligned}$$

$$\begin{aligned}
&-3\psi'' - 5\xi'' - 2\chi'' - \frac{244}{21\eta}\psi' - \frac{1978}{63\eta}\xi' - \frac{718}{63\eta}\chi' - \frac{2672}{1323\eta^2}\psi + \\
&+\left(-\frac{2000}{\eta^{\frac{169}{63}}} - \frac{5344}{441\eta^2}\right)\xi - \frac{5344}{1323\eta^2}\chi + \frac{1}{\eta^{\frac{169}{63}}}\delta N^{(6)} - \frac{1}{\eta^{\frac{169}{63}}}\delta M^{(6)} = 0\,, \qquad \text{(A.8)}
\end{aligned}$$

$$\begin{aligned}
&-\frac{1}{2}\chi'' - \frac{250}{63\eta}\chi' + \frac{9}{7\eta}\psi' + \left(-\frac{170}{49\eta^2} - \frac{6000}{\eta^{\frac{169}{63}}}\right)\chi + \left(-\frac{9000}{\eta^{\frac{169}{63}}} - \frac{43}{49\eta^2}\right)\psi + \\
&+\left(-\frac{510}{49\eta^2} - \frac{15000}{\eta^{\frac{169}{63}}}\right)\xi + -\frac{3}{\eta^{\frac{169}{63}}}\delta N^{(6)} = 0\,. \qquad \text{(A.9)}
\end{aligned}$$

The first equation, containing only first derivatives, is taken as a constraint on the initial data. We are interested in the late time behavior, i.e. $\eta \gg 1$. Keeping only the leading order terms the remaining equations can be approximated as,

$$-2\chi''-2\psi''-6\xi''-\frac{230}{7\eta}\xi'-\frac{176}{21\eta}\psi'-\frac{230}{21\eta}\chi'-\frac{12868}{3969\eta^2}\chi-\frac{6434}{3969\eta^2}\psi-\frac{12868}{1323\eta^2}\xi = 0\,, \qquad \text{(A.10)}$$

$$-3\psi''-5\xi''-2\chi''-\frac{244}{21\eta}\psi'-\frac{1978}{63\eta}\xi'-\frac{718}{63\eta}\chi'-\frac{2672}{1323\eta^2}\psi-\frac{5344}{441\eta^2}\xi-\frac{5344}{1323\eta^2}\chi = 0\,, \qquad \text{(A.11)}$$

$$-\frac{1}{2}\chi''-\frac{250}{63\eta}\chi'+\frac{9}{7\eta}\psi'-\frac{170}{49\eta^2}\chi-\frac{43}{49\eta^2}\psi-\frac{510}{49\eta^2}\xi = 0\,. \qquad \text{(A.12)}$$

This set of coupled equations can be solved exactly and we find the leading terms to be

$$\psi \sim \frac{c_1}{\eta^{0.06}} + \frac{c_2}{\eta^{0.36}}\,, \qquad \text{(A.13)}$$

$$\xi \sim c_3 + \frac{c_4}{\eta^{0.06}}\,, \qquad \text{(A.14)}$$

$$\chi \sim c_5 + \frac{c_6}{\eta^{0.06}}\,. \qquad \text{(A.15)}$$

As a consistency check, we can plug this result back into the original equations (A.6)–(A.9). In addition we must consider the evolution of $\delta N^{(6)}$, $\delta M^{(6)}$ and δM given by (3.17) and (3.18). We find that it was consistent to neglect the time evolution of the string sources (i.e. δN and δM) compared with the expansion of the background and the evolution of the perturbations. In addition, we find that at late times this holds as an exact solution of the perturbation equations. In this way, we find a self consistent solution for the behavior of the long wavelength linear perturbations.

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